CHRIST CHURCH GRAMMAR SCHOOL

12_3CD MAT Examination Mid-Year 2011

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This paper is in two parts: Section A (35 marks) in which calculators may not be used, and Section B (65 marks) in which calculators may be used.

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR Curriculum Council Formula Sheet.

TO BE PROVIDED BY THE CANDIDATE

Standard Items: Pens, pencils, eraser or correction fluid, ruler

Special Items: Drawing instruments, templates and up to 2 calculators satisfying the conditions set by the Curriculum Council, plus one sheet of A4 paper, i.e. 2 pages of A4 notes in the calculator allowed section only.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room.

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INSTRUCTIONS TO CANDIDATES

Show all working clearly, in sufficient detail to allow your answers to be checked readily, and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

SECTION A – CALCULATORS MAY NOT BE USED IN THIS SECTION

Question	Maximum Mark	Mark Achieved
A1	3	
A2	10	
A3	5	
A4	6	
A5	5	
A6	6	
TOTAL	35	

SECTION A – NO CALCULATORS ALLOWED IN THIS SECTION

Question A1 [3 marks]

Simplify (i) $\dfrac{-2x+3}{-3+2x}$

(ii) $\dfrac{20!}{18!}$

(iii) $(x+1)^2-(x-1)^2$

Section A Question 2 [2+2+2+4=10 marks]

(i) Use the quotient rule to differentiate $\frac{2x^2+1}{x-4}$. Simplify your answer if possible.

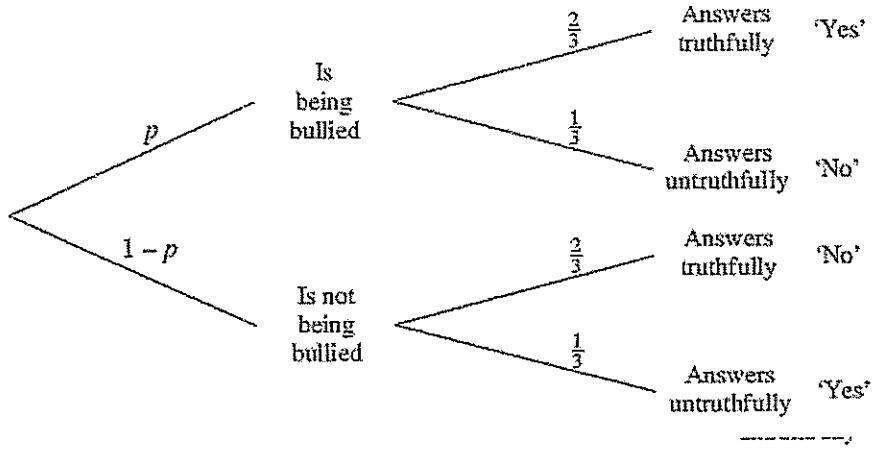
(ii) Find $\frac{dy}{dx}$ if $y=(2e^{3x}-3)^4$ Simplify your answer if possible.

(iii) Find $\int (3x-1)(3x^2-2x)^3\,dx$ Simplify your answer if possible.

(iv) Solve the simultaneous equations $\begin{cases} x+y-z=5 \\ 2x-y+3z=9 \\ -x+2y-2z=-2 \end{cases}$

Section A Question 3 [2+3=5 marks]

A researcher is investigating the proportion p of children who are being bullied at school. To overcome any reluctance children might have to answering questions about being bullied, the following procedure is used. The researcher asks 'Are you being bullied at school?'. Before answering, the child being interviewed throws an unbiased die (unseen by the researcher); if the score on the die is 1, 2, 3 or 4 the child answers the question truthfully and if the score is 5 or 6 the child answers untruthfully. This procedure is illustrated in the tree diagram below.



(i) Show that the probability that a child answers 'Yes' to the researcher's question is $\frac{1}{3}(1+p)$.

(ii) The researcher finds that on average 35% of children answer 'yes'. Find the value of p and hence the probability that a child answers the question truthfully.

Section A Question 4 [2+2+2=6 marks]

Four married couples are to be seated in a row of eight seats numbered 1 to 8.

(i) If men and women alternate, find the number of different ways in which the eight people can be arranged. (Leave your answer in factorial form)

(ii) If the four men sit down in seats 1, 2,7 and 8, find the number of possible seating arrangements for the eight people. (Leave your answer in factorial form)

(iii) Find the total number of arrangements in which the men sit next to their wives.

(Do not leave your answer in factorial form)

Section A Question 5 [1+2+1+1=5 marks]

"The Bat" is batting for Australia in the fifth Test. It is the last ball of the over and Australia needs one run to win the series. "The Bat" hits the ball but it goes straight up in the air. The height of the ball at any time t (seconds) is given by $h(t) = 6t - t^2$ where the height is measured in metres.

(a) Find the height of the ball after 4 s

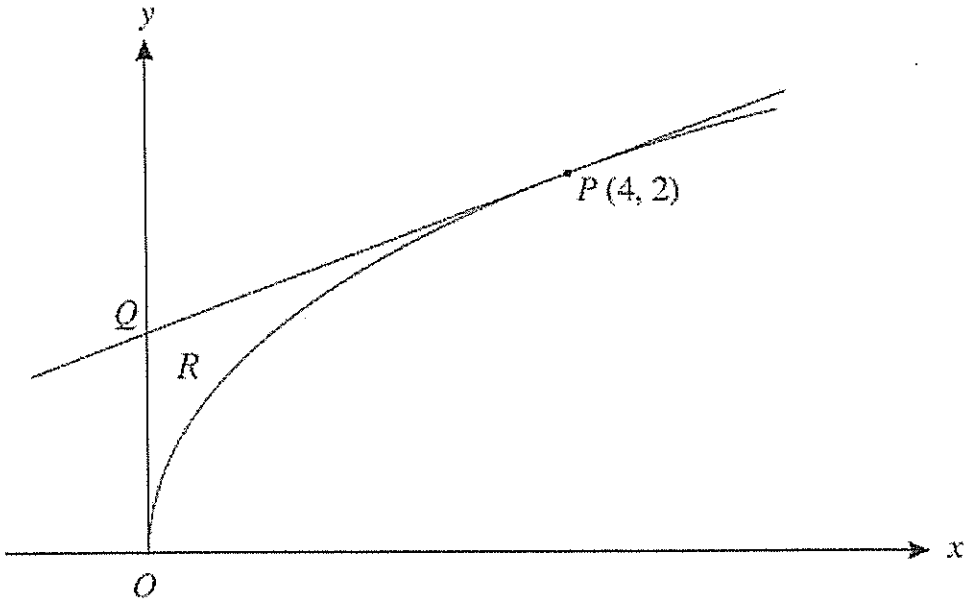
(b) Use a calculus method to find when the ball is at its greatest height.

(c) What is the greatest height reached?

(d) A fieldsman who is standing 20m away from the wicket begins to run in immediately after "The Bat" hits the ball at 4 m/s in an attempt to catch the ball. Will the fieldsman be able to make up enough ground to catch the ball? (show your working)

Section A Question 6 [6 marks]

The diagram shows the graph of $y = \sqrt{x}$



The point P on the graph has coordinates $(4, 2)$. The tangent at P meets the y-axis at Q. Find the exact area of the region R bounded by the curve, the y-axis and the tangent PQ.

End of non-calculator section

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SECTION B – CALCULATORS MAY BE USED IN THIS SECTION

Question	Maximum Mark	Mark Achieved
B7	4	
B8	8	
B9	7	
B10	6	
B11	7	
B12	7	
B13	5	
B14	4	
B15	8	
B16	4	
B17	5	
TOTAL	65	

SECTION B – CALCULATORS MAY BE USED IN THIS SECTION

Section B Question 7 [4 marks]

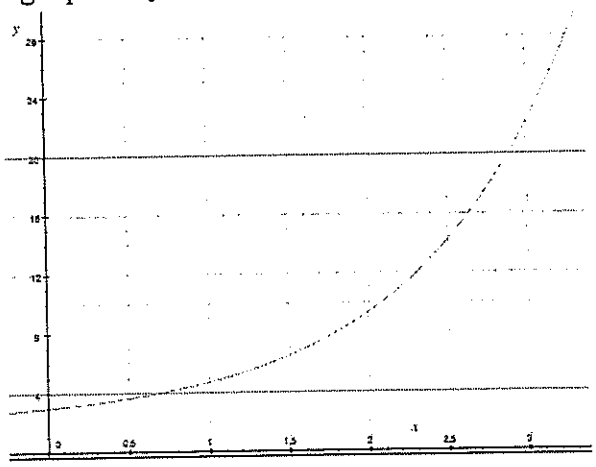
For each of (a) and (b) below, draw the first two stages of the tree diagram that represents the situation.

(a) Steven and Christopher play each other often at one-on-one basketball. They start with an even chance of winning but if Steven wins then his confidence increases and he is 70% likely to win their next game. However, if he loses then he has only a 45% chance of winning the next game.

(b) A golfer is equally likely on the first day of a tournament to wear white, red or blue shoes. She never wears the same colour shoes on successive days but is three times as likely to wear the red shoes after the white shoes as she is to wear the blue shoes. If she has worn the blue shoes she is twice as likely to wear the white shoes next, while her next choices are equally likely if she wears the red shoes.

Section B Question 8 [3+3+2=8 marks]

The graph of $y = e^x + 2$ is shown below.



(a) Find the equation of the tangent to the curve at the point $x = 2$, using exact values.

(b) Find the exact co-ordinates of the x-intercept of this tangent line and draw it on the graph above.

(c) A triangle is formed by the tangent line you have drawn in (b), the x-axis and the line $x = 3$. Find the area of this triangle in exact form.

Section B Question 9 [1+1+1+2+2=7 marks]

If we have the digits 0, 2, 4, 5, 7, and each digit may be used once only,:

(a) How many 5-digit numbers can be formed?

(b) find the number of possible odd numbers greater than 30 000

(c) find the number of possible 5-digit numbers that start and end with an even number.

(d) If all five digits are used what is the probability that such a number is greater than 40000 given that it is greater than 30000?

(e) Three digits are chosen one at a time at random from the 5 digits and not replaced. What is the probability that they are in ascending order?

Section B Question 10 [2+4=6 marks]

A group of students is tested to determine how much information they learn during a lesson. Let x units denote the amount of information presented during a lesson, and let y units denote the amount of information learned by the students.

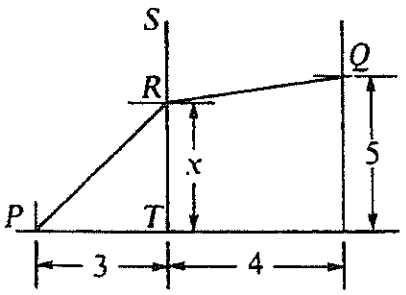
Suppose that $y = xe^{-0.01x}$, $x > 0$

(a) Find $\frac{dy}{dx}$

(b) Find how much information should be presented during a lesson in order to maximise the amount of information learned by the students. If this amount of information is presented, find the proportion of this information that is learned.

Section B Question 11 [3+3+1=7 marks]

A pipeline is to be laid between places P and Q and is to consist of two straight pieces PR, RQ, where R is a point on the fixed line TS.



All distances shown are in kilometres. Due to a variety of factors affecting construction, the cost of construction between P and R, in thousands of dollars, is twice the square of the distance, measured in kilometres, between P and R, while between R and Q it is three times the square of the distance.

(a) Express the total length of pipeline in terms of x in simplest form.

(b) Express the total cost of construction, C thousand dollars, in terms of x in simplest form.

(c) Find the minimum cost of the pipeline.

Section B Question 12 [1+1+2+2+1=7 marks]

The number of virus particles present in a patient can be modelled by the equation $N(t) = 150e^{0.1t}$, where t is the number of hours after observations began.

(a) What is the initial number of virus particles?

(b) Find the number present after 2 hours.

(c) Find the exact rate of increase in virus particles 2 hours after observations began.

(c) Find the time when the rate of increase of virus particles will be 60 per hour.

(e) The number of virus particles of a virus B can be modeled by $N(t) = 50e^{0.25t}$. At what time will the number of virus particles A and B be equal?

Section B Question 13 [5 marks]

A spherical balloon is being inflated so that its surface area is increasing at a constant 12π cm^2 / s

(a) Find the exact rate of change of the radius of the balloon at the time when the surface area is 1600 cm^2.

(b) Find the rate of change of the volume of the balloon at the instant that the surface area is 1600 cm^2.

Section B Question 14 [4 marks]

Two events, A and B have P(A) =0.3 and P(B) = 0.4

(a) If A and B are independent find

(i) $P(A \cap B)$

(ii) $P(\overline{A \cap B})$

(iii) $P(\overline{A \cup B})$

(b) If A and B are not independent, what is the maximum value of $P(A \cup B)$?

Section B Question 15[2+2+2+2=8 marks]

A survey at the local petrol station has revealed that 30% of the customers use diesel, 65% use unleaded, and the remainder use premium-unleaded.
60% of the customers who use diesel fill their tank, 30% of the unleaded users fill their tank and 50% of the premium-unleaded customers fill their tank.

(a) find the probability that a customer will fill up with diesel.

(b) find the probability that a customer does not fill the tank completely.

(c) find the probability that if a customer fills the tank, diesel is used.

(d) find the probability that a customer fills the tank, or uses unleaded, but not both.

Section B Question 16 (4 marks)

The organisers of the "Win a Dinner" eating contest assume that they will get 1 000 entrants if the entry fee is \$50. If the entry fee is increased by \$5, they predict they will lose 25 competitors. To run the contest the organisers have fixed costs of \$20 000. Let x represent each \$5 increase.

(a) Show that the revenue can be expressed as $\$(50\,000 + 3750x - 125x^{2})$.

(b) Find the expression for profit, in terms of x.

(c) How many entries are required to achieve the maximum profit?

Section B Question 17 (5 marks)

A curve with equation $y = ax^4 + bx^3 + cx + d$ has a zero gradient at the point (0,1), and touches, but does not cross, the x-axis at the point (-1,0)

Find the values of a, b, c and d

End of calculator section

THIS PAGE IS AVAILABLE FOR EXTRA WORKING IF NECESSARY. IF YOU USE IT TO REPLACE A PREVIOUS ATTEMPT CROSS OUT THE PREVIOUS ATTEMPT AND MAKE SURE YOU NUMBER YOUR NEW ANSWER CORRECTLY.